

Plasma Heating and Current Drive by LH Waves*

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Dedicated to Professor Dieter Pfirsch on his 60th Birthday

The paper presents a general picture of LH wave propagation and absorption relevant to plasma heating and current drive experiments, which is based on a secular (in space) nonlinear effect due to the ponderomotive density change produced by the LH pump itself. It appears to be a suitable model for observations under a large variety of wave and plasma conditions, in their general trends, relevant scaling and absolute values, although there are no adjustable free parameters in the model. More specifically, the theory provides a unified explanation for the anomalously strong absorption of high-speed “slow” waves (thus solving the so-called LH “spectral gap” problem), observed both in current drive and heating situations, as well as for the existence, parameter scaling and absolute value of the actual density limits.

1. Introduction

LH experiments, performed under a variety of wave and plasma conditions [1–4], have produced an important body of far-reaching results, some of which are still not understood:

1. Waves launched in a narrow high phase-velocity range, expected to be inadequate to strongly interact with enough particles for full absorption, turned out to be waves which can drive the tokamak current with the highest efficiency. This is the so-called “spectral gap” problem.

2. Under certain circumstances, bulk ion heating has been produced at much lower plasma densities than expected from quasi-linear theory for the k_{\parallel} -spectra of the antennae.

3. Current drive, electron and ion heating, and, possibly, just wave propagation into the plasma can each have an upper density limit. They have the same simple frequency dependence, suggestive of an equally simple interpretation [8]. This, however, neither quite reproduces the systematically lower experimental values of the limits, even in the case of hydrogen plasmas, nor provides the observed weak ion-mass dependence for deuterium plasmas (generally, but inconvincingly, attributed to absorption by some hydrogen minority).

Of the various aspects of the LH wave problem – launching, propagation and absorption – it is certainly propagation that is the least clear.

The electromagnetic field in the plasma in front of phased arrays of waveguides can be modelled with high accuracy. A large variety of k_{\parallel} -spectra has been produced. It has been proved, however, that, for example, adding some high k_{\parallel} -tail to the main radiation lobe of one antenna (or using more than one antenna with different k_{\parallel} -spectra), has no bearing upon the problem of broadening the k_{\parallel} -spectrum well inside the plasma (point 1.), since such a tail would rather be depleted at the plasma periphery.

The LH experiments afford no indication that one should distrust the quasilinear description of the wave-induced particle diffusion in velocity space. Of course, typically quasilinear quantities such as the absorbed power density and the generated current density depend very strongly (exponentially) upon the extent of the actual k_{\parallel} -spectrum within the plasma, which is the most elusive piece of information of the whole problem. However, there is at least one important quantity, the incremental efficiency of current generation (i.e. the electron current density generated per unit power density absorbed by the electrons), which has only a weak k -dependence, essentially as $\{\min(k_{\parallel})\}^{-2}$ [5]. This is a fortunate circumstance since $\min(k_{\parallel})$ is one of the few RF quantities which can be considered as known well even inside the plasma. In

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wide density ranges the experiments have fully confirmed this important theoretical prediction.

The treatment of LH wave propagation in the plasma interior has been performed in the linear approximation. The general belief is that, to become substantial in the plasma interior, the nonlinear effects caused by the ponderomotive change produced in the time-averaged density by the LH wave itself, require much larger wave amplitudes than those involved in the experiments under consideration. However, waves launched by antennae of finite extent (along the static magnetic field) have been shown to undergo a secular (in space) ponderomotive effect which is such that even waves with arbitrarily small amplitude are eventually nonlinearly distorted if they penetrate by a sufficient number of radial wavelengths into the plasma [6]. A similar effect was discovered in a somewhat different context involving the complex, modified KdV-equation [7].

This paper gives a general picture of LH wave propagation and absorption which rests on the secular nonlinear effect just referred to. It appears to be a suitable model of the observations in their general trends, relevant scalings and absolute values, particularly with regard to points 1. to 3., although there are no adjustable free parameters in the model.

In conformity with the experimental evidence that the relevant physical effects are largely insensitive to the details of the equilibrium, a relatively crude geometry will be assumed: a cartesian uniform plasma slab (or even a half-space) of period $2\pi R$ in the z -direction (which is along the magnetic field). This allows us to solve the nonlinear partial differential equation to which the radiation-interference problem under consideration can be reduced. The virtual impossibility of recovering the results by means of mode- or ray-superposition techniques should be clear from the fact (see next section) that the dielectric tensor involved in the problem has a δ -like anti-Hermitian peak in the very neighbourhood of a certain singular surface which is determined by the field amplitude and the ray's slope.

2. Qualitative Outline of the Theory

In this section we summarize in a qualitative way the content of the succeeding sections. For sim-

plicity, we consider a uniform plasma half-space. It is well known that in the linear electrostatic approximation the slow wave pattern consists of parallel straight rays which project the boundary values of the RF field amplitude into the plasma (resonance cone). The ponderomotive effect changes the slope of the rays by focusing or defocusing them, depending on the sign of $\partial M/\partial z$, where M is the module squared of the z -component of the electric field. Eventually the convergent rays start building up a caustic surface, the surface where $|\text{grad } M| \rightarrow \infty$. Concomitantly, the production of high $|\mathbf{k}|$ field components grows explosively, resulting in abrupt wave absorption. This is because the high $|\mathbf{k}|$ field components stretch the plateau-like enhancement of the particle velocity distributions down to the thermal domain, which is equivalent to an exponential increase in electron-Landau and ion-Karney damping. This is precisely the required k_{\parallel} -upshift mechanism which is capable of progressively counteracting the liner depletion of the high- $|k_{\parallel}|$ part of the spectrum as the rays propagate from the plasma periphery. The role of abrupt full absorption in determining the whole LH wave picture can hardly be overestimated. If absorption is neglected, the solution of the nonlinear partial-differential equation is multi-valued in the region of space beyond the caustic surface. Indeed the geometry of the rays is such that the caustic surface consists of two branches, extending radially to infinity, which merge in a cusp line (located fairly close to the antenna). Before impinging on the caustic, the rays forming one branch cross the rays forming the other. Such inconsistency is removed altogether by including in the wave propagation equation a δ -like wave energy sink (resulting from a δ -like anti-Hermitian part of the otherwise Hermitian dielectric tensor) centered on some surface $x = x_0(z)$ still to be determined. This energy sink ensures that when the rays pass through it, they bend and fade as, concomitantly, the transported field amplitudes undergo total absorption. The x_0 surface is found to consist of two branches which extend radially to infinity and merge in a cusp line. One of the branches obeys the same differential equation as the Hermitian caustic, while the other branch is a new surface. With a proper choice of the integration constant the cusp line of the x_0 surface can be made to coincide with the cusp line of the Hermitian caustic. Single-valuedness is ensured everywhere simply by

identifying the one branch with the proper Hermitian branch. The choice is such that the rays which, in the Hermitian problem, intercepted the rays forming, eventually, the other branch now die out when impinging on the new branch, which is a bow-wave-like surface (see Fig. 1). The entire discontinuity surface we are left with separates a region of space where $M \neq 0$ from a “downstream” region where $M = 0$.

While the wave propagation and (total) absorption problem can thus be considered as solved, the question of the distribution of the absorbed power between ions and electrons remains open. It requires evaluation of the quasilinear diffusion coefficients in terms of the Fourier transforms in \mathbf{k} -space of the electric field components relative to the situation just described. This can be done to a sufficient extent, but is cumbersome. It will not be carried out in the present paper. Instead, a general remark is made on the expected effect of the form of the \mathbf{k} -spectrum on the heating of ions and electrons with reference to the following, frequently used argument [8].

In the expressions of the power densities absorbed by ions and electrons, P_i and P_e , there is an exponential dependence only on $(\omega/k_\perp^\parallel v_i)^2$ and $(\omega/k_\parallel^\parallel v_e)^2$ (where k_\perp^\parallel and k_\parallel^\parallel are the effective upper boundaries of the k_\perp and k_\parallel ranges of the wave spectrum and $v_\alpha = 2KT_\alpha/m_\alpha$ are the thermal speeds). The equipartition equation $P_i = P_e$, once written as

$$(k_\perp^\parallel v_i/k_\parallel^\parallel v_e)^2 = 1 + \ln F, \quad (1)$$

can thus be directly used to determine with logarithmic accuracy the density for “switch-over” from electron to ion heating. The point is that the function F depends on the various wave and plasma parameters but is such that for most experimental conditions the value of the RHS of (1) is between 1 and 2 [8], and that the dispersion relation is approximately electrostatic, so that the LHS of (1) does contain the equilibrium quantities and the wave frequency but not the wave numbers. Therefore (1) can provide the value of the “switch-over” density even if the actual \mathbf{k} -spectrum in the plasma has to be different (so to speak, “upshifted”) from the antenna spectrum to give substantial absorption. We want to stress, however, that the validity of this density determination rests on the tacit assumption that the \mathbf{k} -spectrum of the LH wave within the

plasma is such as to produce, in the relevant velocity ranges, virtually single-humped quasilinear diffusion coefficients D_{QL}^i and D_{QL}^e . The point here is that the coefficients D_{QL}^i and D_{QL}^e corresponding to a LH pump with a given range of \mathbf{k} 's have different extents along the relevant directions in velocity space. A rectangular \mathbf{k} -spectrum line gives a rectangular D_{QL}^e but results in a D_{QL}^i with an additional high velocity part which decreases as slowly as v_\perp^{-2} [9]. Thus, if the actual \mathbf{k} -spectrum has separate lines, as expected from the narrow-line spectra of antennae with many waveguides, ion heating should be favoured because the ion distribution function would have a continuous plateau-like enhancement over a broad velocity range, while over the corresponding velocity range the electron distribution function would only have a stair-like enhancement. This could explain the deuterium heating experiments in the Petula B tokamak.

So far we have been concerned with the observed anomalously strong absorption of relatively high-speed “slow” waves, which we attribute to the ponderomotive build-up of a singular surface with a Hermitian caustic branch and an anti-Hermitian bow-wave branch. We now want to show briefly that also the existence of density limits, referred to in the introduction, can be directly explained in the framework of the same theoretical model. It is due to the incipient destruction of the discontinuous wave pattern by interference when its extent along the toroidal direction becomes sufficient to allow rays with different slopes to cross. We first consider the case of a unidirectional antenna, as used for current drive, assuming that all the rays are right-going ($k_\parallel > 0$). The LH wave pattern from a finite-length antenna is not only resonance-cone like due to almost electrostatic slow waves. There are also fast electromagnetic waves. The slope (with respect to the direction of the static magnetic field) of the fast rays is always less than that of the slow rays with the same k_\perp -value, and is k_\parallel -dependent. Therefore, even if the antenna radiation lobe is strongly peaked as a function of k_\parallel , the fast rays do not form a well collimated pencil. During the first crossing of the plasma poloidal cross-section, the pencil spreads out poloidally and toroidally and eventually, with the first reflection at the opposite plasma edge, diffuse “eigenmodes” start to build up. We shall show that above a certain density, and for wave frequencies below a certain value, the slow ray

pencil crosses with the fast rays. Then, provided the relevant k_{\parallel} -values are not too high, the superposition of the two kinds of rays destroys the singular surface from the crossing place downstream (to infinity). In a real plasma such a ray crossing excludes the singular wave pattern from a wider and wider central core when the plasma density is increased. As all LH current drive observations consistently point to very peaked radial profiles for the driven current density, such an effect suitably explains the existence of a cut-off density. The latter is given by a condition on $(k_{\perp}/k_{\parallel})$ in terms of the plasma geometry; it has the correct value and m_i scaling.

In the case of nondirectional antennae (with k_{\parallel} -spectra virtually symmetric with respect to $k_{\parallel} = 0$) various kinds of crossings are possible, including crossing of the right-going slow-ray pencil with the left-going slow-ray pencil under proper conditions for both toroidal and poloidal superposition, which can explain the experimental upper density limits for plasma heating.

The succeeding sections of the paper are organized as follows.

In Sect. 3 we briefly derive the relevant nonlinear partial differential equation for the quantity M in the electrostatic absorption-free approximation (this part of the work has already been published [6]). The equation is formally solved by writing the solution as an implicit function, which immediately reveals the existence of a singular surface. The latter is discussed in a representative case. Reference to quasilinear absorption results [10], presented elsewhere, points to the necessity of introducing an energy sink term to ensure consistency. This is carried out in Sect. 4, where a δ -like anti-Hermitian term is added to the RHS of the wave equation considered in the previous section. It is noteworthy that this is sufficient for unique determination of the whole (electrostatic) wave pattern, in particular the position, form and nature of the actual singular surface. With Sect. 5 the treatment of the nonlinear wave problem is interrupted to produce a supplementary piece of information: about the structure of the (electromagnetic) fast wave component of the complete LH radiation pattern. The disruptive effect these waves may have on the singular surface of the nonlinear problem is discussed in Sect. 6, in relation to the existence of the density limits. A tabular comparison with the evidence from relevant

experiments strongly supports the theoretical results. The main results of the paper are summarized in Section 7.

3. Solution of the Absorption-Free Electrostatic Problem

The starting point is the equation $\text{div}(\epsilon \underline{E}) = 0$, where ϵ , the collision-free part of the dielectric tensor [11], depends on $|E_e|^2 \equiv M$ owing to the ponderomotive effect, which modifies the density as $n = n_0(1 - \gamma M)$, where

$$\gamma \equiv \frac{\omega_{pe}^2 + \omega_{pi}^2}{4\pi n_0 \epsilon_{11} (T_i + T_e) \omega^2}. \quad (2)$$

It is assumed that

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t} + \text{cc}$$

and

$$k_{\parallel} E_x - k_{\perp}(M) E_z = 0, \quad (3)$$

which is the electrostatic approximation, with $k_y = 0$, for one wave with given k_{\parallel} , is valid at every point; here $\frac{k_{\perp}^2(M)}{k_{\parallel}^2} = -\frac{\epsilon_{33}(M)}{\epsilon_{11}(M)}$. Moreover, we assume $E_y = 0$ since E_y is zero at the grill mouth. We then get an equation for the component E_z only:

$$\partial_x \left[\epsilon_{11}(M) \frac{k_{\perp}(M)}{k_{\parallel}} E \right] + \partial_z [\epsilon_{33}(M) E] = 0, \quad (4)$$

where, for simplicity, E is written for E_z . By writing

$$\epsilon_{ik}(M) = \epsilon_{ik} - \gamma M \epsilon'_{ik}, \quad (5)$$

where ϵ'_{ik} is the derivative of ϵ_{ik} with respect to the density, one gets

$$\begin{aligned} & -\gamma E \left(\epsilon_{11} \frac{k_{\perp}}{k_{\parallel}} \right)' \partial_x M + \epsilon_{11}(M) \frac{k_{\perp}(M)}{k_{\parallel}} \partial_x E \\ & - \gamma E \epsilon'_{33} \partial_z M + \epsilon_{33}(M) \partial_z E = 0. \end{aligned} \quad (6)$$

By multiplying this equation by E^* and taking the real part of the resulting equation one gets

$$\begin{aligned} & \left[\epsilon_{11} \frac{k_{\perp}}{k_{\parallel}} - 3\gamma M \left(\frac{\epsilon_{11} k_{\perp}}{k_{\parallel}} \right)' \right] \partial_x M \\ & + [\epsilon_{33} - 3\gamma M \epsilon'_{33}] \partial_z M = 0, \end{aligned} \quad (7)$$

which can be written as

$$\epsilon_{11}(3M) \frac{k_{\perp}(3M)}{k_{\parallel}} \partial_x M + \epsilon_{33}(3M) \partial_z M = 0. \quad (8)$$

The characteristics of (8) are the solutions of

$$\frac{dz}{dx} = \frac{k_{\perp}}{k_{\parallel}} \frac{\varepsilon_{33}(3M)}{\varepsilon_{11}(3M)},$$

i.e. of

$$\frac{dz}{dx} = -\frac{k_{\perp}(3M)}{k_{\parallel}}. \quad (9)$$

Since M is constant along the characteristics, the solutions of (9) are

$$z + \frac{k_{\perp}(3M)}{k_{\parallel}} x = \tau, \quad (10)$$

τ being a constant. The solution of (8), which satisfies the condition $M(x=0, z) = M^0(z)$, M^0 being a given function, is then (see, for example, [12])

$$M = M^0(T(x, z)), \quad (11)$$

where $\tau = T(x, z)$ is implicitly defined by (10). Thus M is the solution of the implicit equation

$$M = M^0\left(z + x \frac{k_{\perp}(3M)}{k_{\parallel}}\right), \quad (12)$$

which we already derived in [6]. This equation immediately reveals the nonperiodicity of wave propagation in the x -direction and the existence of a secular effect. A similar effect was discovered in a somewhat different context by Karney [7]. In [6] we remarked, too, that $|\text{grad } M|$ is infinite along the curve (caustic), which is a surface in the (x, y, z) space, where

$$1 + M^0(\tau) 3\gamma \frac{k'_{\perp} x}{k_{\parallel}} = 0. \quad (13)$$

The equation for the caustic is best represented in the parametric form

$$x_c = -[3\gamma M^{0'}(\tau) k'_{\perp}/k_{\parallel}]^{-1},$$

$$z_c = \tau + \frac{k_{\perp}(3M^0(\tau))}{k_{\parallel}} [3\gamma M^{0'}(\tau) k'_{\perp} k_{\parallel}]^{-1} \quad (14)$$

(it is recalled that k'_{\perp} is the derivative of k_{\perp} with respect to density) derived from (10) and (13).

To within a form factor, x_c coincides with the self-focusing distance derived in [7], which is independent of thermal dispersion (thermal dispersion, which is disregarded in the present paper, was included in the wave equation of [7]).

Where $M^{0''}(\tau) = 0$, one has $dx_c/d\tau = 0$. For the same τ -values one has $dz_c/d\tau = 0$ because (9) must be satisfied and $k_{\perp}(3M)$ is finite. The curve $(x_c(\tau), z_c(\tau))$ thus has a cusp where $M^{0''}(\tau) = 0$.

For a single-humped M^0 , a reasonable assumption, the caustic has the form sketched in Fig. 1; τ_0 is the smallest value for which $M^{0''}(\tau) = 0$. M is defined for all z 's only when $x < x_c(\tau_0)$, the formal solution of (8) being many-valued beyond the caustic.

Evaluation of the power absorbed by ions or electrons requires a knowledge of the RF diffusion coefficient, i.e. the Fourier transform of the electric field. We derived it in [6] for the region $x < x_c(\tau_0)$. The power absorption was then evaluated numerically. This was reported in [10]. The conclusions were that strong power absorption takes place in the very neighbourhood of the caustic, the reason being that, if at some point $|\text{grad } E|$ goes to infinity, the Fourier transform of E decreases only as $1/k$ when $k^2 \rightarrow \infty$ (so that the RF diffusion coefficients decrease only as $1/v^2$ when $v^2 \rightarrow 0$). This k -dependence relatively increases the amount of energy in the slowest (high k) waves which interact with the plasma bulk, thereby enhancing plateau formation in the velocity distribution functions and increasing the number of particles in the tails very effectively.

4. Solution of the Electrostatic Problem with Energy Sink

The equation $\text{div}(\epsilon \underline{E}) = 0$, where the power absorption is neglected, can be used to evaluate the RF diffusion coefficient and thereby the power absorption only if the absorption coefficient is small. In our case, however, there is strong absorption, which is localized in the neighbourhood of the caustic. Moreover, beyond the caustic the solution of (4) is no longer single-valued, as has already been remarked. All of this leads to us to introduce absorption explicitly in the wave equation, in the form

$$\text{div}(\epsilon \underline{E}) = \beta E_z \delta(x - x_0(z)). \quad (15)$$

The determination of β and $x_0(z)$ is based on the following arguments:

At the caustic power absorption leads to a step-like form of \underline{E} . The general form of (15) is the obvious result of the presence in the dielectric

tensor of an "effective collision frequency" very localized in space. An electric field proportional to a step function is subject to yield jump conditions along $x = x_0(z)$. It is easy to see that, if ε_{12} is neglected, the jump conditions which follow from Maxwell's equations are

$$\begin{aligned} [E_x] \frac{dx_0}{dz} + [E_z] &= 0, \\ [\varepsilon_{11} E_x] - [\varepsilon_{33} E_z] \frac{dx_0}{dz} &= \beta E_z. \end{aligned} \quad (16)$$

These two equations can be satisfied only if one allows for the presence of the two possible electrostatic rays, the left-going and the right-going rays. In other words, the ray which propagates into the plasma from the antenna, let it be right-going, couples to a left-going ray at the curve x_0 . Let us call E^+ the field of the injected ray and E^- the left-going field; moreover, let $x_{1,2}$ be the two branches of x_0 (see Figure 1). Along the caustic the absorption of E^+ is total. We identify dx_1/dz with the derivative of the equation of the caustic (see (13)); (16) thus determines β and the jump of E^- . On the second branch, x_2 , (16) determines dx_2/dz and the jump of E^- . We then have to check that the two branches x_1 and x_2 are consistent, i.e. we have to show that the characteristics which impinge on one branch do not intersect the other branch before, at variance with what happens with the caustic (see Figure 1). We now proceed to the detailed calculations.

From the first of (16) one gets for the branch x_1 with $dx_1/dz = dx_c/dz$

$$[E^-]_c \left(1 - \Pi(M) \frac{dx_c}{dx} \right) = -[E^+]_c \left(1 + \Pi(M) \frac{dx_c}{dz} \right), \quad (17)$$

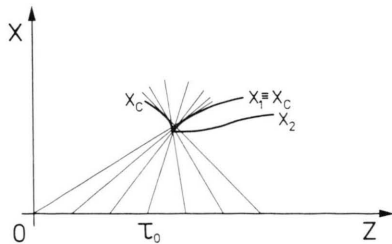


Fig. 1. Caustic, x_c , and the two branches $x_{1,2}$ of the curve x_0 . The straight lines are the characteristics

$$z - \left| \frac{k_{\perp} (3M^0(\tau))}{k_{\parallel}} \right| x = \tau.$$

with

$$\Pi(M) = \frac{k_{\perp}(M)}{k_{\parallel}} = E_x^+/E_z^+,$$

and where $[\]_c$ indicates a jump across x_c . By noticing that

$$1 + \Pi(M) \frac{dx_c}{dz} = O(\gamma)$$

$(\Pi(3M) - \Pi(M))/\Pi = O(\gamma)$ on the caustic (see (14)), one has

$$[E^-]_c = -\frac{1}{2} \left(1 + \Pi(M) \frac{dx_c}{dx} \right) [E^+]_c. \quad (18)$$

The second of (16) then yields

$$\begin{aligned} \beta E^+ &= -\varepsilon_{11}(M) \Pi(M) \left(1 + \Pi(M) \frac{dx_c}{dz} \right) E^+ \\ &\quad - \varepsilon_{11} \Pi \left(1 - \Pi \frac{dx_c}{dz} \right) [E^-]_c \end{aligned} \quad (19)$$

or

$$\beta = -2\varepsilon_{11} \Pi \left(1 + \Pi(M) \frac{dx_c}{dz} \right) \quad (20)$$

$$= -4\varepsilon_{11} \gamma M \Pi', \quad (21)$$

Π' being the derivative of Π with respect to density. On the curve x_2 the equations have the same form as on x_1 ; now, however, β is given by (21) and the jump is across x_2 . The equation corresponding to (19) is

$$\begin{aligned} \beta E^+ &= \varepsilon_{11}(M) \Pi(M) \left(1 + \Pi(M) \frac{dx_2}{dz} \right) E^+ \\ &\quad - \varepsilon_{11} \Pi \left(1 - \Pi \frac{dx_c}{dz} \right) [E^-]_2. \end{aligned} \quad (22)$$

The equation corresponding to (18) is

$$[E^-]_2 = - \left(1 + \Pi(M) \frac{dx_2}{dz} \right) \frac{E^+}{2}, \quad (23)$$

so that one gets

$$1 + \Pi \frac{dx_2}{dz} = \frac{\beta}{2\varepsilon_{11} \Pi} \quad (24)$$

or

$$\frac{dx_2}{dz} = -\frac{1}{\Pi} \left(1 + 2\gamma M \frac{\Pi'}{\Pi} \right) = -\frac{1}{\Pi(2M)}. \quad (25)$$

The parametric representation of $x_2(z)$ follows from the conditions

$$z = \tau - x_2(\tau) \Pi(3M^0(\tau)) \quad (26)$$

and

$$(dx_2/d\tau)/(dz_2/d\tau) = \frac{dx_2}{dz}. \quad (27)$$

One gets

$$\begin{aligned} & (dx_2/d\tau) [\Pi(2M^0(\tau)) - \Pi(3M^0(\tau))] \\ & - x_2(d\Pi(3M^0(\tau))/d\tau) + 1 = 0, \quad (28) \\ & z = \tau - x_2 \Pi(3M^0(\tau)). \end{aligned}$$

It is easy to see that $(dx_2/d\tau) = 0$ implies $(dz_2/d\tau) = 0$; hence $x_2(z)$ has a cusp. The position of the cusp follows from the second of (28):

$$1 - x_2(d\Pi(3M^0(\tau))/d\tau) = 0. \quad (29)$$

A comparison with (13) shows that the values of τ which yields a cusp are the same as for the caustic. We choose the solution of the differential equation for x_2 such that $x_2(\tau_0) = x_c(\tau_0)$, so that the cusps of x_2 and x_c coincide. We then define $x_1(z)$ as the part of the caustic $(x_c(\tau), z_c(\tau))$ which corresponds to $\tau \leq \tau_0$, i.e. we identify x_1 with the upper branch of the caustic. The second branch of x_0 is the lower part of $x_2(z)$, i.e. $(x_2(\tau), z_2(\tau); \tau > \tau_0)$ (see Figure 1). This choice is consistent in the sense previously

stated, because at every point $\left| \frac{dx_2}{dz} \right| < \left| \frac{dx_c}{dz} \right|$, so that for every z one has $x_c \geq x_2$. It is also the only choice which is consistent, as is easy to see. By using realistic profiles for $\gamma(x)$ and $M^0(z)$, the distance of the cusp from the plasma surface is usually found to be a small fraction of the plasma minor radius.

A consequence of the previous results is that energy propagates along the characteristics and impinges on the curves x_1 and x_2 , where it is absorbed. Absorption thus takes place along the whole curves x_1 and x_2 , mostly on the last curve. A small fraction of the injected power, of the order of γ , is transferred from the right-going ray into a left-going one at x_1 and x_2 . In this way strongly localized absorption is consistently explained within the model.

Evaluation of the power distribution between ions and electrons would require evaluation of the RF diffusion coefficients, and hence of the Fourier transform of the electric field. This is not done here.

5. Fast Wave Radiation Pattern

So far we have described the LH waves in the electrostatic approximation. In the following we want to show that the electromagnetic field which accompanies the electrostatic rays interacts with them when they wind around the torus more than one turn before reaching the magnetic axis. This interaction inhibits the secular nonlinear effect, hence wave absorption and eventually causes the so-called “density limit”. We anticipate that the density limit is related by this mechanism to the equation $2\pi R/r_p = k_\perp/k_\parallel$, R and r_p being the major and minor radii of the torus, and

$$\frac{k_\perp}{k_\parallel} = \frac{\omega_p}{\omega} \left[1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2} \right]^{-1/2}.$$

A thorough discussion follows in Section 6.

We first derive the field excited by a LH antenna in the plasma, in a form suited to our purpose. We again assume uniform density and no dependence on the y -coordinate and place the antenna at $x = 0$. Although this is a somewhat crude model, it is sufficient to give a reasonable estimate of the distribution and magnitude of the field due to the fast waves.

Maxwells equations with $\partial_y = 0$ are

$$\begin{aligned} \partial_{zz} E_x - \partial_{zx} E_z &= -\frac{\omega^2}{c^2} (\epsilon_{11} E_x + \epsilon_{12} E_y), \\ \partial_{zz} E_y + \partial_{xx} E_y &= -\frac{\omega^2}{c^2} (\epsilon_{21} E_x + \epsilon_{22} E_y), \\ \partial_{xz} E_x - \partial_{xx} E_z &= \frac{\omega^2}{c^2} \epsilon_{33} E_z. \end{aligned} \quad (30)$$

They are satisfied by

$$\begin{aligned} E_i &= E_i^F + E_i^S = \int_{-\infty}^{\infty} \tilde{E}_i^F(k) e^{ikz} e^{i\sigma_F(k)x} dk \\ &+ \int_{-\infty}^{\infty} \tilde{E}_i^S(k) e^{ikz} e^{i\sigma_S(k)x} dk, \end{aligned} \quad (31)$$

where $i = x, y, z$ and

$$\begin{aligned} \sigma_F^2 &\sim \frac{\omega^4}{c^4} |\epsilon_{12}|^2 \frac{1}{(k^2 - \epsilon_{11} \omega^2/c^2)}; \\ \sigma_S^2 &\sim \frac{\epsilon_{33}}{\epsilon_{11}} \left(\epsilon_{11} \frac{\omega^2}{c^2} - k^2 \right) \end{aligned} \quad (32)$$

are the Fast and Slow solutions of the dispersion relation

$$D = \varepsilon_{11} \sigma^4 + [(\varepsilon_{33} + \varepsilon_{11})(k^2 - \varepsilon_{11} \omega^2/c^2) - \varepsilon_{12}^2] \sigma^2 + \varepsilon_{33} [(k^2 - \varepsilon_{11} \omega^2/c^2) + \varepsilon_{12}^2] = 0. \quad (33)$$

The signs of σ_F and σ_s have to be taken so as to satisfy the radiation condition. The quantities E_i^F and E_i^s are determined, as usual, from the external field by the continuity conditions at $x = 0$

$$[E_y] = [E_z] = 0, \quad [B_y] = [B_z] = 0,$$

i.e.

$$\begin{aligned} \tilde{E}_y^F + \tilde{E}_y^s &= \tilde{E}_y^0, \\ \tilde{E}_z^F + \tilde{E}_z^s &= \tilde{E}_z^0, \\ k(\tilde{E}_x^F + \tilde{E}_x^s) + \sigma_F \tilde{E}_z^F + \sigma_s \tilde{E}_z^s &= \tilde{B}_y^0, \\ \sigma_F \tilde{E}_y^F + \sigma_s \tilde{E}_y^s &= -\tilde{B}_z^0, \end{aligned} \quad (34)$$

where \tilde{E}_i^0 , \tilde{B}_i^0 are the Fourier transforms of the antenna field at $x = 0$. The four conditions (34) are redundant since the polarization of \underline{E} is determined by the dielectric tensor. We take the first two equations, with $\tilde{E}_y^0 = 0$ and \tilde{E}_z^0 given, a choice corresponding well to the experimental situation. We introduce for short the notation

$$\frac{\tilde{E}_y^F}{\tilde{E}_z^F} \equiv \Pi_y^F = -\frac{\varepsilon_{12}}{k \sigma_F} \frac{\sigma_F^2 - \varepsilon_{33}}{\sigma_F^2 + k^2 - \varepsilon_{11}} \quad (35)$$

and analogously for \tilde{E}_y^s/E_z^s . It then follows that

$$\tilde{E}_z^F = \frac{-\Pi_y^F}{\Pi_y^F - \Pi_y^s} \tilde{E}_z^0, \quad \tilde{E}_z^s = \frac{\Pi_y^F}{\Pi_y^F - \Pi_y^s} \tilde{E}_z^0, \quad (36)$$

where $(\Pi_y^F - \Pi_y^s)$ is proportional to $\sqrt{k - \xi}$, $\xi \equiv \sqrt{\varepsilon_{11} + \varepsilon_{12}/\sqrt{-\varepsilon_{33}}}$ being the value of k for which $\sigma_s^2 = \sigma_F^2$. For x larger than a few perpendicular wavelengths the contribution from $|k| < \xi$ is obviously negligible since then σ_s^2 and σ_F^2 are complex and yield evanescent waves, so that we can substitute the intervals $(-\infty, -\xi)$ and (ξ, ∞) for the intervals $(-\infty, \infty)$ in the integrals of (31).

To discuss the asymptotic form of these integrals (the (x, z) region where the integrals become asymptotic will become clear in the following), we use the fact that (see e.g. [15]) the most important contribution comes from the neighbourhood of $k = \pm \xi$ and from the neighbourhood of the points of stationary phase, if they exist.

For the first contribution we have to evaluate integrals of the form (see (31) and (36))

$$F = \int_{\xi} e^{i k z} e^{i(a \pm b \sqrt{k - \xi})} \frac{dk}{\sqrt{k - \xi}}, \quad (37)$$

where $(a \pm b \sqrt{k - \xi})$ is the expression for σ_s and σ_F when $k \sim \xi$; here

$$a^2 \equiv |\varepsilon_{12}| \sqrt{-\varepsilon_{33}/\varepsilon_{11}}$$

and

$$b^2 \equiv 8(\varepsilon_{33} + \varepsilon_{11})^2 \xi |\varepsilon_{12}| \sqrt{-\varepsilon_{11}/\varepsilon_{33}}.$$

One gets

$$\begin{aligned} F &= \frac{2}{\sqrt{-iz}} e^{i(\xi z + ax)} \exp\{-i b^2 x^2/4z\} \cdot \\ &\cdot \left[\sqrt{\pi} - \int_{-\infty}^{\mp b x/2 \sqrt{-iz}} e^{-y^2} dy \right] \\ &= -i \frac{e^{i(\xi z + ax)}}{\sqrt{-iz}} [2i \sqrt{\pi} \exp\{-i b^2 x^2/4z\} \\ &\quad - Z(\mp b x/2 \sqrt{-iz})], \end{aligned} \quad (38)$$

where Z is the plasma dispersion function. This yields

$$E_z = \lim_{k=\xi} \left(\frac{\sqrt{k - \xi}}{\Pi_y^F - \Pi_y^s} \right) \Pi_y^F \tilde{E}_z^0(\xi) [F(b) - F(-b)]. \quad (39)$$

The interesting region is $b x/\sqrt{|z|} \gg 1$; there, although E_z^F and E_z^s behave as $1/\sqrt{|z|}$, one gets

$$E_z \sim -\frac{4i}{bx} e^{i(\xi z + ax)} \lim_{k=\xi} \left(\frac{\sqrt{k - \xi}}{\Pi_y^F - \Pi_y^s} \right) \Pi_y^F \tilde{E}_z^0(\xi). \quad (40)$$

In order to evaluate the contribution from the points of stationary phase, we apply the usual method in a somewhat modified form, as follows: We have to evaluate

$$\int e^{i(kz + \sigma_s x)} \exp\{\ln \tilde{E}_z^s\} dk.$$

The derivative with respect to k of the exponent is zero when

$$((\tilde{E}_z^s)'/\tilde{E}_z^s) + i(z + \sigma_s' x) = 0. \quad (41)$$

Obviously, the most important contribution to the integral comes from the neighbourhood of the maximum of \tilde{E}_z^s , i.e. since $(\Pi_y^F/(\Pi_y^F - \Pi_y^s)) \sim 1$, from the neighbourhood of $k = k_0$ if $[d\tilde{E}_z^0/dk]_{k=k_0} = 0$. Along the line $z + \sigma_s' x = 0$ (41) is then satisfied

at $k = k_0$, and one gets

$$E_z^s \sim \tilde{E}_z^s(k_0) e^{i(k_0 z + \sigma_s(k_0) x)} \cdot \int \exp \{ (i \sigma_s'' x + (\tilde{E}_z^{s''}/\tilde{E}_z^s)) (k - k_0)^2/2 \} dk, \quad (42)$$

i.e.

$$E_z^s \sim \tilde{E}_z^s(k_0) e^{i(k_0 z + \sigma_s(k_0) x)} \cdot \sqrt{2\pi} (i \sigma_s'' x + \tilde{E}_z^{s''}/\tilde{E}_z^s)^{-1/2}. \quad (43)$$

Thus the “slow” field component propagates along the line $z + \sigma_s' x = 0$ and decreases as $1/\sqrt{x}$ as soon as

$$x \sim \tilde{E}_z^{s''}/\tilde{E}_z^s \sigma_s'' \sim L^2 k_0^3 \frac{c^2}{\omega^2} (-\epsilon_{11} \epsilon_{33})^{-1/2},$$

as a consequence of the spread of the ray; here L is the half-width of the antenna. In the electrostatic approximation one gets $\sigma_s'' = 0$; then E_z^s does not spread and its modulus remains constant along the line $z + \sigma_s' x = 0$.

The field E_z^F can be evaluated analogously. \tilde{E}_z^F is approximately given by $-\tilde{E}_z^0 \Pi_y^s/\Pi_y^F$, i.e. by

$$-\tilde{E}_z^0 \frac{\epsilon_{12}^2}{\epsilon_{33}} \frac{\sigma_F \omega^2}{\sigma_s k^2 c^2}. \quad (44)$$

It can be shown that far from the cutoff of σ_F the solution of

$$\frac{\partial}{\partial k} \ln \tilde{E}_z^F = 0 \quad (45)$$

is approximately the same as the solution of

$$\frac{\partial}{\partial k} \ln \tilde{E}_z^0 = 0.$$

The solution of (45) is such that \tilde{E}_z^F goes to zero when the density approaches the cutoff value for σ_F . This means that the “fast” ray spreads out when they approach the cutoff region from the high-density side.

A difference with respect to E_z^s is that there is no approximation where σ_F' is negligible. A further difference is that E^F is expected to be much less localized poloidally than E^s because the last field is virtually electrostatic. In cylindrical geometry the decrease of the field along the rays is counterbalanced by the factor $(r_p - x)^{-1/2}$ (r_p is the plasma minor radius).

Finally, the ratio of E_z^F and of E_z^s along their respective rays is

$$\left| \frac{E_z^F}{E_z^s} \right| \sim \left| \frac{\tilde{E}_z^F}{\tilde{E}_z^s} \right| \sqrt{\frac{\sigma_s''}{\sigma_F''}} \sim \frac{\Pi_y^s}{\Pi_y^F} \sqrt{\frac{\sigma_s''}{\sigma_F''}} \sim \frac{1}{\sqrt{2}} \left(\frac{\epsilon_{11} \epsilon_{12}^2}{\epsilon_{33}} \right)^{3/4} \frac{1}{N^4}. \quad (46)$$

Note the strong dependence on N , the parallel index of refraction.

6. The Density Limit Problem

The “slow” rays which leave the antenna, essentially one for every lobe of the antenna spectrum, have the same slope in the electrostatic approximation, whereas the “fast” rays have different slopes, as follows from

$$\begin{aligned} |\sigma_s'| &\sim \sqrt{\frac{-\epsilon_{33}}{\epsilon_{11}}} \frac{N}{(N^2 - \epsilon_{11})^{1/2}}, \\ |\sigma_F'| &\sim |\epsilon_{12}| \frac{N}{(N^2 - \epsilon_{11})^{3/2}}, \end{aligned} \quad (47)$$

$$\begin{aligned} \epsilon_{11} &= 1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\Omega_e^2} \sim 1 + n \left(\frac{10.3}{B^2} - \frac{4.38}{A f^2} \right), \\ |\epsilon_{12}| &\sim \frac{\omega_{pe}^2}{\omega \Omega_e} \sim 287 \frac{n}{f B}, \end{aligned} \quad (48)$$

$$|\epsilon_{33}| \sim \frac{\omega_{pe}^2}{\omega^2} \sim 8.06 \cdot 10^3 \frac{n}{f^2} \quad [\text{GHz tesla } 10^{14} \text{ cm}^{-3}].$$

The quantities $|\sigma_s'|$ and $|\sigma_F'|$ decrease as the density increases, so that when the density is large enough, the “slow” ray makes more than a toroidal turn before reaching the magnetic axis, and can intersect the “fast” ray pencil, which is not strongly poloidally localized. This occurs at

$$\begin{aligned} x_0 &= 2\pi R / (|\sigma_s'| - |\sigma_F'|), \\ z_0 &= 2\pi R |\sigma_F'| / (|\sigma_s'| - |\sigma_F'|). \end{aligned} \quad (49)$$

In the region where the rays intersect there is no caustic and the absorption mechanism we proposed in Sect. 4 is destroyed, as we now show.

In the intersection region the electric field is

$$\underline{E} = \underline{E}^s(z + \sigma_s' x) + \underline{E}^F(z + \sigma_F' x), \quad (50)$$

so that for $M = |E_z|^2$ one gets

$$M \sim |E_z^s|^2 + 2 \text{Re} \{ (E_z^s)^* E_z^F \} \quad (51)$$

since $|E_z^F| \ll |E_z^s|$. The slow dependence $1/\sqrt{x}$ is irrelevant in this context, so that the curves along which M is constant can be written as

$$z = -\sigma_s'(3M)x + \xi + \Delta, \quad (52)$$

where ξ is a free parameter and Δ goes to zero with $|E^F|$. Along the curves (52) one can write

$$M \sim |E_z^s(\xi)|^2 + \Delta \frac{\partial}{\partial \xi} |E_z^s(\xi)|^2 + 2 \operatorname{Re} \{ (E_z^s(\xi))^* E_z^F(\xi + k_0 x (\sigma'_s + \sigma'_F)) \} \quad (53)$$

so that

$$\Delta(x, \xi) = - \frac{2 \operatorname{Re} \{ (E_z^s)^* E_z^F \}}{\frac{\partial}{\partial \xi} |E_z^s|^2} + f(\xi), \quad (54)$$

where f is chosen such that $\Delta = 0$ on the curve where the intersection begins.

For the new caustic one gets the parametric representation

$$\begin{aligned} -x_c \frac{\partial}{\partial \xi} (\sigma'_s) + 1 + \frac{\partial \Delta}{\partial \xi} &= 0, \\ z_c - \xi + x_c \sigma'_s - \Delta &= 0. \end{aligned} \quad (55)$$

One thus has

$$\frac{\partial}{\partial \xi} z_c = -\sigma'_s \frac{\partial}{\partial \xi} x_c + \frac{\partial \Delta}{\partial \xi} \frac{\partial}{\partial \xi} x_c,$$

and hence along the caustic

$$\frac{dz_c}{dx} = -\sigma'_s + \frac{\partial \Delta}{\partial x}. \quad (56)$$

It is now recalled that β has the sign of $-(1 + \Pi(M) dx_c/dz)$ (see (20)). With the new caustic one gets

$$\beta_{\text{new}} \sim \beta + \frac{\partial \Delta}{\partial x}. \quad (57)$$

Thus β_{new} changes sign as long as $|\partial \Delta / \partial x| \gtrsim |\beta|$, i.e. as long as

$$\left| \frac{E_z^F}{E_z^s} \right| \gtrsim \gamma M, \quad (58)$$

or, from (46)

$$(\varepsilon_{12}^2 / \varepsilon_{33})^{3/4} \frac{1}{N^4} > \gamma M. \quad (59)$$

By using (2), we get for γM the estimate

$$\gamma M \sim 3.74 \cdot 10^{-4} N \Phi / f^2 (T_i + T_e) \sqrt{\varepsilon_{11} n}, \quad (60)$$

where Φ is the energy flux of the antenna in kW/cm², f the frequency in GHz and T_x in keV. Equation (59) is then satisfied if

$$N \lesssim 6.9 \left(\frac{n}{B^3} \right)^{1/4} \left(\frac{f^2 (T_i + T_e)}{\Phi} \right)^{1/5}. \quad (61)$$

For today's experimental situations this value lies between 2.5 and 5 and covers the values of N used for current drive.

Owing to the physical meaning of β , the absorption described in Sect. 4 cannot take place in the intersection region. Moreover, since the only assumption in solving (15) is that (E_x/E_z) is independent of k , i.e. that E is electrostatic, it can be concluded that through the nonlinear interaction of E^s and E^F the field is no longer essentially electrostatic. Thus the absorption mechanism of Sect. 4, which is due to the electrostatic nature of the field, is not possible beyond the intersection. No power is absorbed in the region $x_0 \leq x \leq r_p$, and if x_0 is large enough no current is driven. The value of the density for which the intersection takes place at $x = r_p$ is given by

$$|\sigma'_s| - |\sigma'_F| = 2\pi R/r_p. \quad (62)$$

We show that (62) cannot be satisfied when the ratio ω/Ω_e is large enough, then there is no "density limit", a conclusion in accord with the experiments. Since $|\sigma'_F| \ll 2\pi R/r_p$, we consider $|\sigma'_F|$ as a given quantity. Equation (62) can now be solved for ω_{pe}^2/ω^2 :

$$\frac{\omega_{pe}^2}{\omega^2} = \frac{((2\pi R/r_p) + |\sigma'_F|)^2}{1 - ((2\pi R/r_p) + |\sigma'_F|)^2 \left(\frac{\omega^2}{\Omega_e^2} - \frac{m_e}{m_i} \right)} \quad (63)$$

It is evident that when

$$\frac{\omega^2}{\Omega_e^2} > \frac{m_e}{m_i} + \left(\frac{2\pi R}{r_p} + |\sigma'_F| \right)^{-2}, \quad (64)$$

(63) cannot be satisfied. As an example we consider the Petula B tokamak ($B = 2.75$ tesla) for $f = 1.3$ GHz and $N = 2.3$; and for $f = 3.7$ GHz and $N = 2.7$ or 1.7. For the first frequency value we have $|\sigma'_F| = 4.15$ when $n = 2 \cdot 10^{13} \text{ cm}^{-3}$ (we take $\varepsilon_{11} = 1$ for simplicity); (64) thus leads to $f^2 > 8.57$, which is not satisfied. For $f = 3.7$ GHz and $N = 2.7$ the results are

$$|\sigma'_F| = 0.96, \quad f^2 > 10;$$

for $f = 3.7$ GHz and $N = 1.7$ one gets

$$|\sigma'_F| = 3.69, \quad f^2 > 8.8.$$

The inequalities for the higher frequency are satisfied, so that we do not expect a "density limit"

as for the lower frequency; these results are in accord with the experiments.

The presence in the antenna spectrum of an additional important lobe corresponding to left-going rays (e.g. in FT) causes an intersection at a density smaller than that given by (62). Indeed, it occurs at the density given by

$$x_0 = 2\pi R(|\sigma'_s| + |\sigma'_F|)^{-1}. \quad (65)$$

This effect is present with every symmetric spectrum. The density intervals corresponding to values of x_0/r_p between 1 and 0.8 for various experiments with asymmetric spectra are listed in Table 1. The theoretical values agree well with the experimental data for the “density limit” of the driven current not only for hydrogen but also for deuterium plasmas. The result of ASDEX that the “density limit” in a deuterium plasma is not as well defined as in a hydrogen plasma is also recovered.

Table 1 also lists the values of the density corresponding to x_0/r_p equal to 1, 0.8 and 0.5 for experiments with symmetric spectra (see (65)). The last density value corresponds to interaction taking place only in the outer plasma region.

Table 1. Density interval for $1 < x_0/r_p < 0.8$ for different experiments. In case of symmetrical spectra also the value of density for $x_0/r_p = 0.5$.

	N	A	n [10^{14} cm^{-3}]
ASDEX,	2	1	0.14 – 0.23
$f = 1.3 \text{ GHz}$,	3	1	0.13 – 0.2
asym. [4]	2	2	0.15 – 0.3 *
	3	2	0.15 – 0.3
Petula B,	2	2	0.14 – 0.28
$f = 1.3 \text{ GHz}$,			
asym. [4]			
Petula B,	3	2	0.1 – 0.165 – 0.48
$f = 1.3 \text{ GHz}$,	> 4	2	0.15 – 0.23 – 0.6 **
sym.			
FT, $B = 4 \text{ T}$ [13]	1.5	2	0.15 – 0.23
	2	2	0.28 – 0.41
FT, $B = 8 \text{ T}$ [13]	1.5	1	0.2 – 0.3
	2	1	0.25 – 0.4
	1.5	2	0.2 – 0.3
	2	2	0.3 – 0.44
Alcator C, $B = 8 \text{ T}$,	1.5	1	2.8 – 6.1
asym. [14]	1.5	2	never
Alcator C, $B = 8 \text{ T}$,	3	1	1.2 – 1.85 – 4.1
sym. [14]	3	2	1.3 – 2.2 – 5.8

* For $0.15 < n < 0.3$ one has $1 < x_0/r_p < 0.9$; for $n > 0.3$ x_0/r_p increases again.

** Evaluated by intersection of the “slow” rays.

For N larger than allowed by (61) the destructive interaction of the “slow” rays with the “fast” ones does not take place. However, the symmetric “slow” rays intersect after a toroidal turn if their poloidal extent is large enough. The result would again be that power is absorbed only in a plasma region which moves outwards as the density increases. This “density limit” does not depend on N , since σ'_s is approximately N -independent. The values reported in Table 1 for Petula B, $N > 3$, are based on these considerations. The detailed calculations will be presented elsewhere.

7. Conclusions

The purpose of the paper was to provide a theoretical model capable of explaining the LH heating and current drive experiments in their general trends, relevant parameter scaling and absolute values. We have found a unified explanation for the anomalously strong absorption of high speed “slow” waves (the “spectral gap” problem) that occurs in both current drive and plasma heating experiments in a large variety of waves and plasma conditions, as well as for the existence and characteristics of the actual density limits.

We have been guided by the experimental evidence that such phenomena are largely insensitive to the details of the equilibrium, and that they are unlikely to rely upon any occasional “outside” effects such as fluctuations at the plasma edge or instabilities. These would require specifying arbitrary parameters whose validity is uncertain. Accordingly, a relatively crude geometry was assumed. On the contrary, consistency required special care in the treatment of power absorption. In the very neighbourhood of a certain surface in space, the otherwise Hermitian dielectric tensor has a strong anti-Hermitian part. This singular surface has two branches which merge in a cusp line: a caustic-like branch and a bow-wave like branch. Such a surface stems from a secular (in space) nonlinear effect due to the ponderomotive density change inherent in LH wave propagation. Quite remarkably, the whole radiation-interference problem is described by one single nonlinear partial differential equation with proper boundary conditions, which can be solved. When the rays approach

the caustic/bow-wave surface, the electric field gradients tend to infinity. It is the concomitant explosive growth of the high-wave-number field components that results in a abrupt total absorption of the wave on such a surface. The question of the distribution of the absorbed wave power between ions and electrons, as a function of the characteristics of the wave-number spectrum of the antenna, has also been discussed, with reference to a frequently used simple argument.

Finally, we have addressed the problem of the existence of density limits for current drive and

particle heating. We have shown that the limitation is due to the disruption by interference of the singular surface, when its toroidal extent becomes sufficient to allow it to cross with rays having a different slope. In the case of unidirectional antennae, which have been discussed in special detail, these are fast rays. During the first crossing of the plasma poloidal cross-section, above a certain density value, the fast rays do indeed cross the slow-ray pencil. A tabular comparison with evidence of the density limit from many experiments strongly supports the theoretical results*.

Note added in proof: In a paper just appeared [16], the current-drive density limit is related to the hot-plasma linear mode coalescence condition for a k -value upshifted ad-hoc to $/v$, since the authors postulate that, as soon as such a condition is fulfilled, perpendicular ion damping outdoes parallel electron damping. This assumption, however, is not correct. The condition for the transition from parallel electron damping to perpendicular ion damping, which is given by Eq. (1) of the present paper, at the mode coalescence corresponds to the condition given in [8], apart from the presence of a factor of 2. Thus the introduction of the thermal correction made by the authors of [16] does not eliminate the inconsistency with the experimental conditions they attribute to [8]. The corrected transition condition one can derive from [16], as the one of [8], lead to higher densities than the limit found (where it exists) in the present paper.

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